Learn to Speak Weinberg!

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Notation

Ω: The null matroid, the unique matroid on the empty set.

G ctr S: For a graph G and edge set S, the graph formed by contracting all edges not in S. See **contraction**.

$G \cdot S$: The **reduction** of a graph G to an edge set S, which is the smallest subgraph containing the edges of S.

$G \times S$: This is $(G \text{ ctr } S) \cdot S$. This means removing the isolated vertices of G ctr S, which equate to connected components in G containing no edge of S.

$\mathcal{P}(G)$: The cycle matroid of G, or just the set of cycles of G. Referred to as the **polygon matroid**.

$\mathcal{B}(G)$: The cut matroid of G, or just the set of bonds of G. Referred to as the **bond matroid**.

$J(b, e)$: The circuit formed by e in b. For a basis b and $e \notin b$, this is the unique circuit such that $e \in J(b, e) \subseteq b \cup \{e\}$.

$(e/e')S$: For $e' \in S$, $e \notin S$, the set obtained by replacing e' with e in S: $(S - \{e'\}) \cup \{e\}$.

$\alpha(S)$: Cardinality of S.

$\overline{S}$: Complement of S.

$r(\mathcal{M})$: **Rank** of a matroid (cardinality of each basis). May be applied to a graph as well, as detailed below.

$\mu(\mathcal{M})$: **Nullity** of a matroid (cardinality of each cobase). May be applied to a graph as well, as described below.

$\mathcal{C} \times S$: For a set $\mathcal{C}$ of circuits, this is $\mathcal{C} \cap 2^S$, the circuits contained in S.

$\mathcal{M} \times S$: For a matroid $\mathcal{M} = (\mathcal{C}, E)$, this is the matroid $(\mathcal{C} \times S, S)$, the **contraction** of $\mathcal{M}$ to S. This can also be used to denote the contraction of a vector space (defined below).

$\mathcal{C} \cdot S$: For a set $\mathcal{C}$ of circuits, this is the collection of minimal intersections of circuits with S.

$\mathcal{M} \cdot S$: For a matroid $\mathcal{M} = (\mathcal{C}, E)$, this is the matroid $(\mathcal{C} \cdot S, S)$, the **reduction** of $\mathcal{M}$ to S. This can also be used to denote the reduction of a vector space (defined below).

$||f||$: The support of a vector f, the set of indices at which its entries are nonzero.

$\mathcal{C}_{\mathcal{V}}$: The set of supports of elementary vectors in the vector space $\mathcal{V}$ on the index set E.

$\mathcal{M}_{\mathcal{V}}$: The matroid $(\mathcal{C}_{\mathcal{V}}, E)$ using the above notation. The **matroid associated with** $\mathcal{V}$.

R_f: Given a vector f, the row vector representation of it.

$R(S)$: For a matrix R with columns indexed by E and $S \subseteq E$, this denotes the submatrix consisting of the columns indexed by S.

$\eta(e, v)$: Given a directed graph, this takes an edge e and a vertex v and returns 0 if they aren't incident, 1 if v is the positive end of e, and -1 if v is the negative end of e.

$\perp \mathcal{V}$: The orthogonal complement of $\mathcal{V}$.

Terminology

base: A basis. Similarly, bases are referred to as **bases**.
binary matroid: A matroid associated with a binary vector space.
binary vector space: A vector space over $\mathbb{F}_2$.
bond: A minimal cut-set. As defined in the paper, a bond of a graph G is a set of edges S such that $G \times S$ is a bond graph.
bond graph: A graph with two vertices and all of its edges between them.
bond matroid: The cut matroid, whose circuits are bonds/cut-sets. Denoted $\mathcal{B}(G)$.
circuit formed by: For a basis b and $e \notin b$, the circuit formed by e in b is the unique circuit C such that $e \in C \subseteq b \cup \{e\}$. It is denoted $J(b, e)$.
cobase: The complement of a basis.
coboundary: For an oriented graph G, this is a vector f indexed by $E(G)$ with $f(e) = \sum_{v \in V(G)} \eta(e, v) g(v)$ for some vector g indexed by $V(G)$. This is the image of the transpose of the signed incidence matrix given in the definition of

1-cycles. Finally, note that these vectors correspond to voltage drops satisfying Kirchhoff's voltage law (where g corresponds to the voltage function on vertices).
coboundary space: The space of coboundaries.
coforest: A maximal edge set containing no bonds.
cographic matroid: A matroid which is the cycle matroid of some graph.
complementary orthogonal space: Orthogonal complement.
contraction of a graph: Given a graph G and set of edges S, the contraction G ctr S is defined by considering the graph H formed by removing the edges in S from G. Then the vertices of G ctr S are the connected components of H and the edges are the edges of S, drawn between the components containing their ends in G. Intuitively, this is the result of contracting all edges except those in S.
contraction of a matroid: For a matroid $(\mathcal{C}, E)$ and $S \subseteq E$, $\mathcal{C} \times S$ is the collection of circuits contained in S. Then $\mathcal{M} \times S = (\mathcal{C} \times S, S)$ is the contraction of $\mathcal{M}$ to S.
contraction of a vector space: For a subspace $\mathcal{V}$ of F^E and $S \in E$, the contraction $\mathcal{V} \times S$ is the subspace of all vectors whose entries are zero at indices not in S. This definition is such that $\mathcal{M}_{\mathcal{V}\times S} = \mathcal{M}_{\mathcal{V}} \times S$.
cotree: An edge set containing no bonds.
1-cycle: Given an oriented graph G, a vector on $E(G)$ over F (a function $f : E(G) \to F$) such that for every vertex v of G, $\sum_{e\in E(G)} \eta(e, v)f(e) = 0$. That is, at each vertex, the sum of the incident function values (respecting orientation) is 0. This can also be viewed as an element of the kernel of the $|V(G)| \times |E(G)|$ matrix whose entries are given by the $\eta(e_i, v_j)$—the signed incidence matrix. Finally, note that these vectors correspond to currents satisfying Kirchhoff's current law.
1-cycle space: The vector space of 1-cycles. I guess this goes without saying.
dual matroid: Bruno and Weinberg define the dual matroid in terms of circuits: given a matroid $\mathcal{M} = (\mathcal{C}, E)$, the dual $\mathcal{M}^*$ has as circuits minimal sets which are orthogonal (as sets) to every member of $\mathcal{C}$. They state without proof that this definition is equivalent to the one in terms of bases.
elementary vector: Given a vector space $\mathcal{V}$ and $f \in \mathcal{V}$, f is elementary if it is nonzero and minimal with respect to inclusion of support: there is no $g \in \mathcal{V}$ with $||g|| \subset ||f||$.
ends: The ends of an edge are the two vertices incident to it, which may be the same.
forest: A (maximal) spanning forest.
graphic matroid: A matroid which is the bond matroid of some graph.
matroid associated with $\mathcal{V}$: Given a vector space $\mathcal{V}$ on an index set E, this is the matroid $\mathcal{M}_{\mathcal{V}} = (\mathcal{C}_{\mathcal{V}}, E)$, where $\mathcal{C}_{\mathcal{V}}$ is the set of supports of elementary vectors in $\mathcal{V}$.
minor of a matroid: Given a matroid $\mathcal{M}$, some matroid of the form $(\mathcal{M}\cdot S)\times T$; that is, some matroid obtained by reduction and contraction.
nullity: Nullity of a graph $\mu(G)$ is the number of edges in each coforest. Nullity of a general matroid $\mu(\mathcal{M})$ is the cardinality of every cobase.

null matroid: The matroid with no elements or circuits. Denoted Ω.
oriented graph: A directed graph. (Strictly, a graph in which each edge has a "negative" end and a "positive" end.) The orientation is given by a function $\eta(e, v)$ which is 0 if v isn't an end of e and ± 1 with appropriate sign if it is. In pictures of these oriented graphs, the arrow goes from the positive to the negative end.
orthogonal: For vectors and vector subspaces, this is defined normally; two sets S and T are orthogonal if $|S \cap T| \neq 1$.
planar matroid: A matroid which is the bond matroid of some graph and the cycle matroid of some graph; that is, both graphic and cographic.
polygon: A cycle within a graph.
polygon graph: A cycle graph.
primitive vector: An elementary vector of $\mathbb{R}^E$, all of whose entries are ± 1 or 0.
rank: Rank of a graph $r(G)$ is the number of edges in each spanning forest; rank of a general matroid $r(\mathcal{M})$ is the cardinality of every basis.
reduction of a graph: Given a graph G and a subset S of its edges, the reduction $G \cdot S$ is the smallest subgraph with those edges.
reduction of a matroid: For a matroid $\mathcal{M} = (\mathcal{C}, E)$ and $S \subseteq E$, $\mathcal{C}$ is the collection of minimal intersections of circuits with S. Then $\mathcal{M} \cdot S = (\mathcal{C} \cdot S, S)$ is the reduction of $\mathcal{M}$ to S.
regular matrix: A matrix with rank equal to the number of rows, and such that every maximal minor is $\pm k$ or 0, for some fixed k.
regular matroid: The matroid associated with a regular vector space.
regular vector space: A vector space over $\mathbb{R}$ such that, for every elementary vector, there is a primitive vector (all entries ± 1 or 0) with the same support.
representative matrix of $\mathcal{V}$: Given a vector space $\mathcal{V}$, a matrix whose rows form a basis of $\mathcal{V}$.
representative vector: Given a vector in function form, this is the corresponding representation as a row vector, denoted R_f.
standard representative matrix: For a representative matrix R and cobase $\bar{b}$ of a vector space $\mathcal{V}$, the matrix $R' = R(\bar{b})^{-1}R$ is the standard representative matrix of $\mathcal{V}$ with respect to $\bar{b}$. This is set up so that $R'(\bar{b})$ is the identity matrix. (The required inverse exists because of Theorem 2.2-2, pg. 74.)
support: The support of a vector is the set of indices at which its entries are nonzero. The support of f is denoted $||f||$ for some reason.
totally unimodular: A matrix is totally unimodular if every minor is ± 1 or 0.
tree: A forest.
Type BI: Given the vector space $\mathcal{V} \subseteq \mathbb{F}_2^7$ with basis vectors $(1,0,0,1,0,1,1)$, $(0,1,0,1,1,0,1)$, $(0,0,1,0,1,1,1)$, the matroid $M_{\mathcal{V}}$ is said to be of Type BI.
Type BII: The dual of a matroid of Type BI. Among binary matroids, these two are a complete set of forbidden minors for the regularity property.
Type HI: The bond matroid of K_5.

Type HII: The bond matroid of $K_{3,3}$. Among binary regular matroids, these two are a complete set of forbidden minors for the property of being a cycle matroid.
Type KI: The cycle matroid of K_5.
Type KII: The cycle matroid of $K_{3,3}$. Among binary regular matroids, these two are a complete set of forbidden minors for the property of being a bond matroid.
valence: Degree. (Loops are counted twice.)
vector on E over F: A vector indexed by E with entries in a field F. This is viewed as a function $f : E \to F$.
vector space on E over F: A subspace of F^E.